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Triangular Proportional Scheme and Concept of the Two Serbian Medieval Churches

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Abstract. Serbian medieval architectural heritage is notable for its sacred architecture including numerous Christian Orthodox churches built at the territory of former Raška state during the period from the 12th to the 14th centuries. Built in the so-called Raška architectural style, characterized by overlapping features of Romanesque and Byzantine traditions, two monuments - the church of the Mother of God in Studenica monastery complex and the church of Holy Dormition in Žiča monastery complex, are exquisite sacred structures and cultural monuments of exceptional importance. They are also remarkable due to geometric-proportional regularities of their design. This paper highlights the importance of geometric concept in relation to proportional analysis of these two structures. Here presented study is conducted by two means: first one – by investigating geometric scheme with equilateral triangles, incorporated into the layout and cross section patterns of each church structure; the second one – by classical proportioning that includes proportional roots, golden section and numeric ratios.

Keywords: Medieval Churches, Proportional Scheme, Equilateral Triangle.

1 Introduction

One of the oldest theoretical questions in the history of architecture is about proportioning systems used in cultural heritage monuments. The design process in building and constructing churches developed over time, while master builders often relied on previous experiences and knowledge [1]. The lack of textual sources about knowledge of medieval architects and other reliable documentation prompts researchers to follow the roots of geometry and its application, far back in the ancient world and geometric exploration of Pythagoras, Plato, or Vitruvius, to whom we owe the first geometric and proportional rules applied in construction of spatial structures [2]. Since the first geometricians were mostly philosophers their overall knowledge affected instructions for the practical proportioning of built structures. In such context, relations between the nature, mathematics (especially geometry) and building were unbreakably bound-

ed, sometimes in the simplest way where measuring units corresponded to the parts of a human body ("feet", "thumb", "elbow", etc). Basic geometric shapes, such as circle, square, or regular polygons were often symbolically related to a human figure and proportioning. Renaissance period brought both Leonardo da Vinci's and Cesare Cesariano's interpretations of Vitruvian man – ideally proportioned human figure which became a matter of numerous discussions. While Leonardo's drawing of a man inscribed in circle and square was connected with golden section rules, which Murtinho disputed in 2016 [3], Evers and Thoenes recognized Cesariano's Vitruvian man (based on triangular construction) as reflection of medieval architectural practice [4]. There is a logical reason for such opinion, because Cesariano investigated proportions of Gothic cathedrals by applying *ad triangulum* principles [5]. However, geometric-architectural reasoning which highlights regular triangular scheme in Christian Orthodox churches design has survived up today [6], hence attributing its overarching character.

This paper investigates proportions of two Serbian medieval churches, which are related to the so-called *Raška* architectural school, by using two geometric means: regular geometric shapes – circle and equilateral triangle, and classical proportioning that included proportional roots, golden section and numeric ratios.

1.1 Historical frame and architectural concept of the two churches

The influence and contribution of Serbian Nemanjić dynasty was crucial for the development of medieval church architecture in former Raška state (currently located in the central part of Serbia). During the period between the second half of 12th until the first half of 14th centuries many churches were built with similar style characteristics, as single nave domed structures usually with three bays and with wide semicircular apses on the east side [7]. Although *Raška* school is like symbiosis of the *east-Byzantine* and *west-Roman* traditions, in majority of cases the influence of Byzantine tradition was perceived as being dominant regarding general structural concept [8]. By expanding upon the work by Štambuk [9] and Tomas [10] the authors question influences of both traditions with respect to proportions of two churches in the monasteries of Studenica and Žiča.

Both churches are under protection of the Institute for the protection of cultural monuments of Serbia as the monuments of exceptional importance. The church of the Mother of God in Studenica, an exquisite example of monumental sacred architecture, was built under local ruler Stefan Nemanja as his legacy - tomb church (built 1186-1205), while the church of the Holy Dormition of the Mother of God in Žiča monastery, later the seat of the archbishop, was built for the purpose of coronation of Nemanja's son Stefan (built 1206-1217).

Studenica's church has a rectangular shape in layout and unique interior architectural space divided by pilasters into three bays. A single nave domed structure with triform altar geometry, square central core and rectangular narthex has asymmetrically positioned side vestibules, the latter of which was the novelty in Serbian medieval architectural program (Fig.1a). The architects of the church in Žiča introduced several changes as well: larger altar space, more opened central core and side chapels con-

nected to the narthex. Further design program of the complex church structures built in Arilje, Davidovica, or Gradac, was traced by such architectural concept. (Fig.1b)

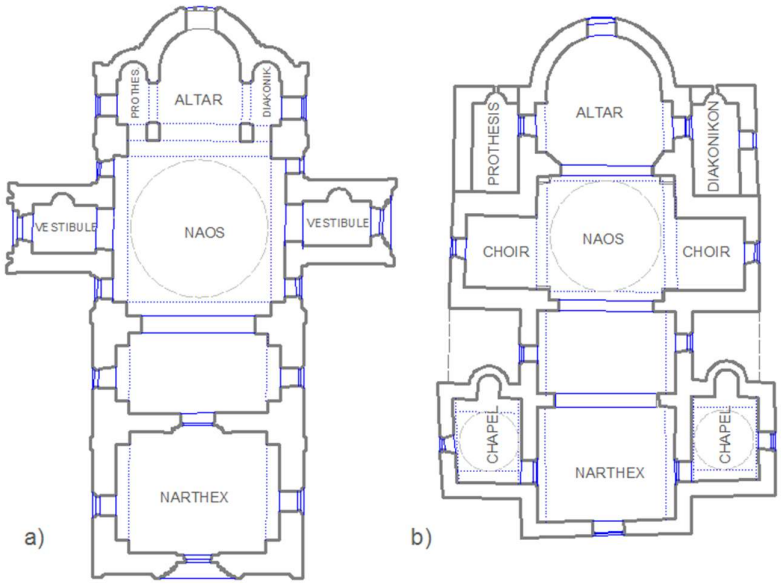

Fig. 1. Architectural program of the two churches: Studenica – a); Žiča – b)

The elegancy of the interior space, established in Studenica's design, remained in Žiča as well, because harmony of the spatial composition was conducted by proportions of their structural elements (Fig. 2).

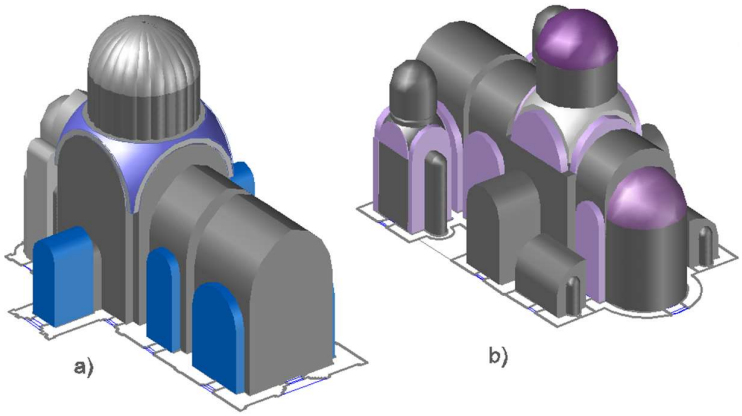

Fig. 2. 3D models of the interior spaces of two structures: Studenica – a), Žiča – b)

2 Proportional analysis

2.1 Documentation sources

Contemporary research on "reverse engineering", i.e. 3D modeling of historical objects and structures utilized laser scanning and photogrammetry as reliable techniques for obtaining accurate technical data [11]. The latest results of proportional investigations of Gothic cathedrals confirmed significance of such accuracy [12]. Following the streamline of new measurement technique applications, results of the terrestrial laser scanning (TLS) performed at Studenica site are used for the current study.

Detailed and reliable documentation (drawings, descriptions and proportional analysis) for both selected churches were found in the monograph studies of Čanak Medić, Kandić and Bošković. The relevance of measuring units present in medieval Raška state is of the importance for the research which includes modular grid [13,14]. Point cloud data of the church exterior obtained by TLS in 2017 confirmed documentation accuracy. Layouts and sections of the churches in Studenica and Žiča show significant and high level performance of masons during building process, with respect to the other churches built on principles of Raška school. The accuracy of building techniques in medieval Serbia at the time varied depending on uncertainty of measuring tools, units and applied materials. Churches that were built of carved stone, especially marble (such as the one in Studenica), were significantly more accurate in obtaining right angles and parallel walls [15]. Annotation and drawings on point cloud deliverables provided for the church exterior in Studenica show neglectable deviation from regular rectangular geometry (Fig. 3).

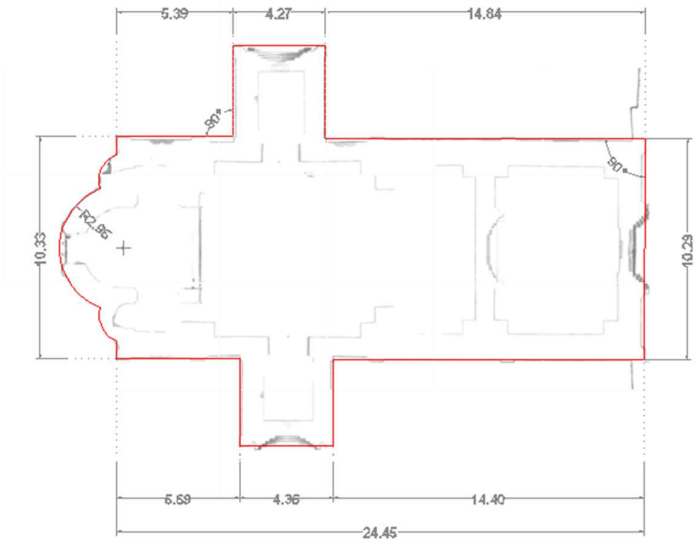

Fig. 3. Representation of the parallelism and orthogonality of walls in Studenica (point cloud)

This accuracy opens the possibility to explore deeper connection of regular geometric shapes, such as circle, equilateral triangle and square in overall proportional system. By expanding upon the latest research concerning general compliance related to I. Štambuk's geometric construction [8] (three dominant Serbian medieval styles/schools are analyzed in [16,17]), geometric analysis is applied in layouts and sections of the Raška school architectural concept patterns.

The other aspect of proportions involves the common ratios applied in the ancient *antique* traditions (1:1; 1:2; 1:√2; 1:√3; golden mean - 1:1.61, etc.). Some of them are related to the proportional rectangles or proportional roots, derived from regular shapes: square, hexagon and pentagon (Fig. 4).

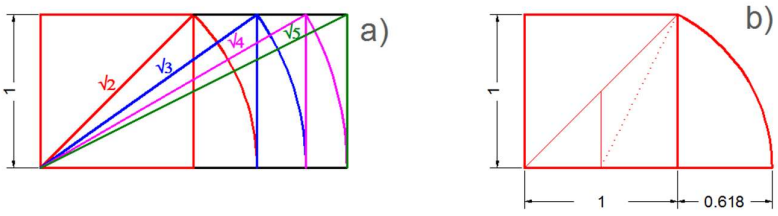

Fig. 4 Graphic representation: proportional roots – a); golden section – b)

Previous investigations [17], where specifications of structural elements were provided for both churches, continued in the form of parallel analysis of their proportions (in layouts and sections). For each functional unit/space inside architectural program, geometric approximation of proportions is provided.

2.2 Proportional analysis guided by regular polygons

Previous testing of the architectural pattern of Raška style [17] on Štambuk's *proportional canon* (two circles and equilateral triangle in strict geometric constraints), addressed to design and proportioning of the early roman churches, has shown several crucial results significant for current exploration:

- key points for *holy triangle*-equilateral triangle (***ABC***) positioning in the church layout and cross section are centers of the *apse* and the *dome*, respectively;
- the *holy triangle* defines interior/exterior width of the church nave;
- ratio 1:√3 (regularity of I. Štambuk's construction) plays an important role in dimensioning of the altar and central space; it appears in proportioning of additional spaces beside the church nave (*pastophoria*, *choir* and *chapels*);
- ratio of the exterior rectangular frame **1234** is 1:√3 (Fig.5a)
- position and dimensions of the key structural element - dome structure along with supportive arches is in compliance with *proportional canon* (Fig.5b);

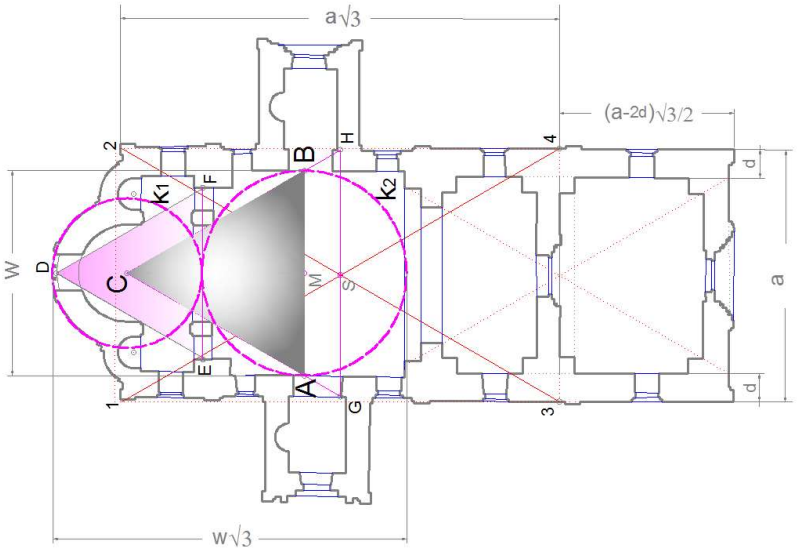

Fig. 5a. Layout of Studenica's church with *proportional canon* [17]

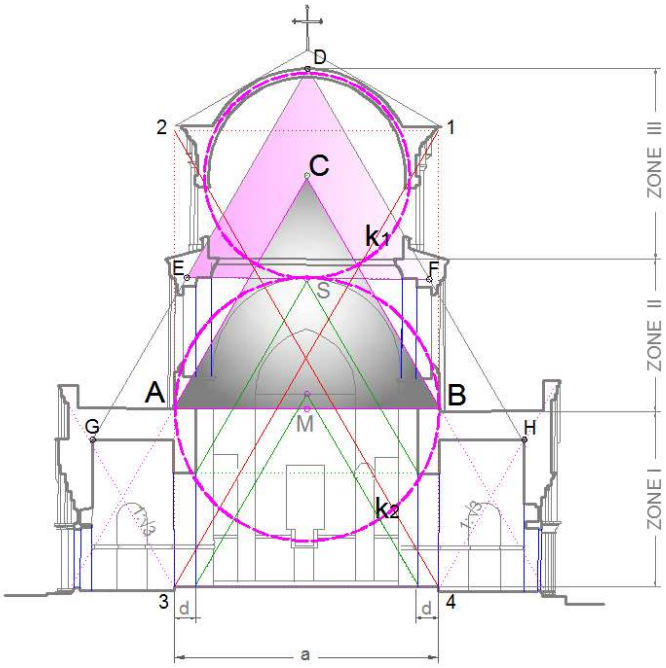

Fig. 5b. Cross section of Studenica's church with *proportional canon* [17]

Comparative analysis of the layouts and sections of the two churches here presented points to strong similarity of geometric approach in their proportioning and design. The most important place of the church in liturgical sense is the altar table, which is in the center of the sanctuary. Geometric scheme introduced by Štambuk accents this point as crucial by setting the vertex ***C*** of the *holy* triangle at the center point of the *apse* (the position of the altar table). The following triangular construction starts from point ***C*** while establishing the relation of the basic – *holy* triangle and the length of the church nave.

In case of the layout of Studenica, the extension of the triangle ***CAB*** to ***CGH*** resulted in constraints with rectangle ***1234***. By mirroring the shape of triangle ***CGH***, shape of a rhomb appears as underlining geometry of the central part of the church. Additional triangle on the west end is of the same size as ***CAB*** (Fig.6a). Since the triangle ***CAB*** in the layout of Žiča is related to the width of the exterior nave, geometric scheme becomes more regular, considering double mirroring of the basic triangle along longitudinal church axis (Fig.6b).

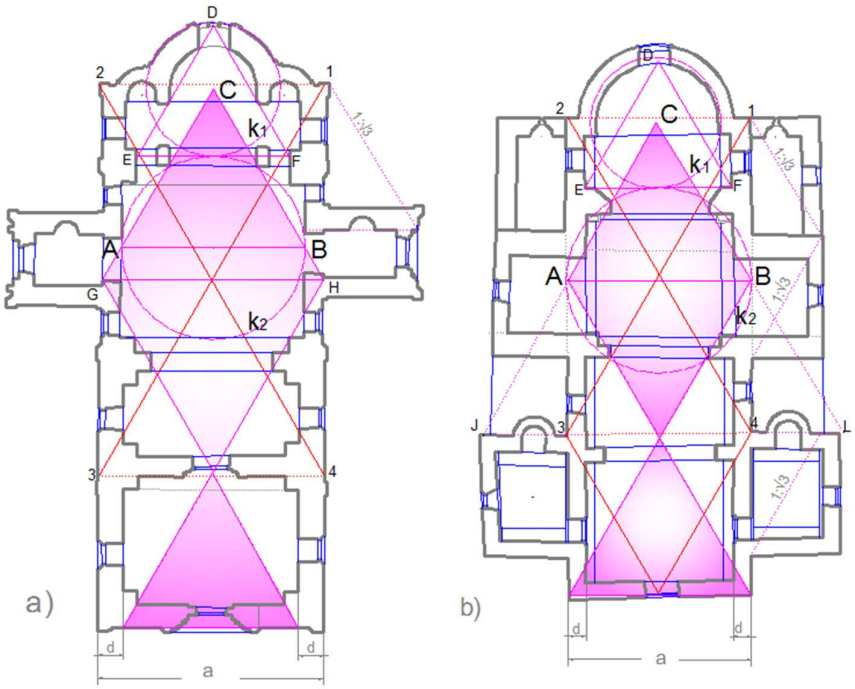

Fig. 6. The *holy triangle* and proportioning scheme: layouts of Studenica – a) and Žiča- b)

An idea of inscribing regular polygons in circles in order to set the proportions of building components in Gothic architecture, as stated by Bork, brought a variety of geometric constructions [18]. While setting regular polygonal scheme, architectural composition has a framing and division role. The contours of walls (both in the interi-

or and exterior) or axes fit into the guiding geometric scheme [1]. Geometric key points have an important role in such dimensioning of objects. This section of the paper presents triangular scheme where guiding elements are top points of the apses and wall contours of the two churches which coincide with triangles vertexes and edges respectively. Dimensioning/measuring of the principal spaces along the longitudinal axes of the church is obtained by mirroring a single equilateral triangle (Fig.7).

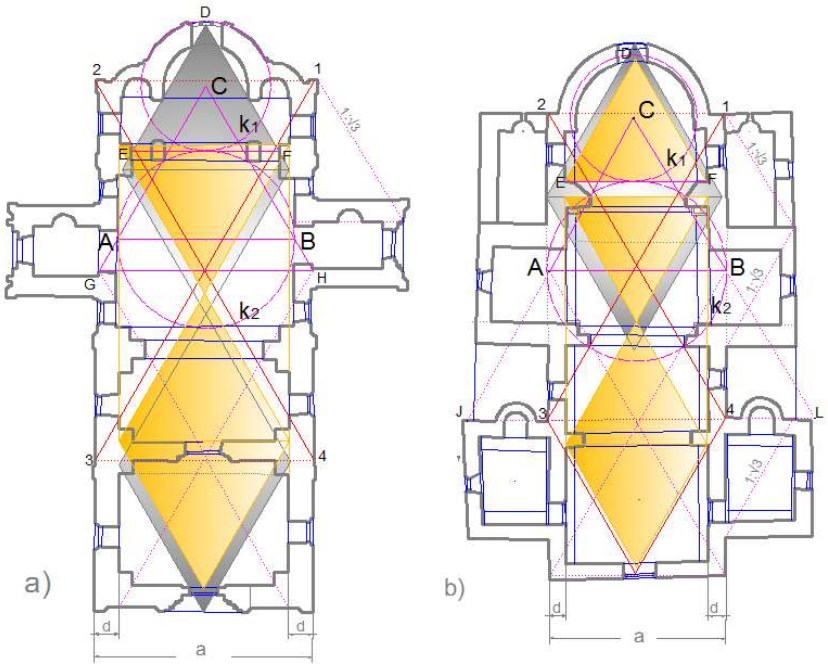

Fig. 7. *Mirroring geometry* of equilateral triangles: Layouts of Studenica – a) and Žiča- b)

Interior division between *narthex*, *naos* and *altar* spaces is obtained by three equilateral triangles (depicted in orange color) starting from the interior edge of the west entrance wall in both layouts. Basic triangle edge is equal to the width of the nave interior (***a-2d***). Mirroring geometry of triangles ends exactly at the edge of the altar's inner wall. However, total length in case of Studenica's layout is *measured* by four triangles (depicted in gray color), where the first and the last ones' edge coincide with spatial division. The difference between two geometric schemes is probably due to the proportioning of the altar space. Dimensioning of joined altar and the central space in Žiča is conducted by exterior width of the nave (***a***), where the total length is $\boldsymbol{a\sqrt{3}}$ (Fig.7).

Similar principles are applied in geometric schemes of the cross sections. Dimensioning of the central core, along with the dome structure is based primarily on in-

scribing the basic triangle into contours of the drum and the dome. Basic triangle edge is equal to the nave interior width (***a-2d***). Central space of Žiča perfectly corresponds to the scheme of three triangles set along the vertical axis of the structure, while obtaining mirroring geometry (Fig.8a). Although slightly different proportions of the lower part of the central core of Studenica appear (caused by corner pilasters), triangular geometry is harmonized with the width of central arches (Fig.8b).

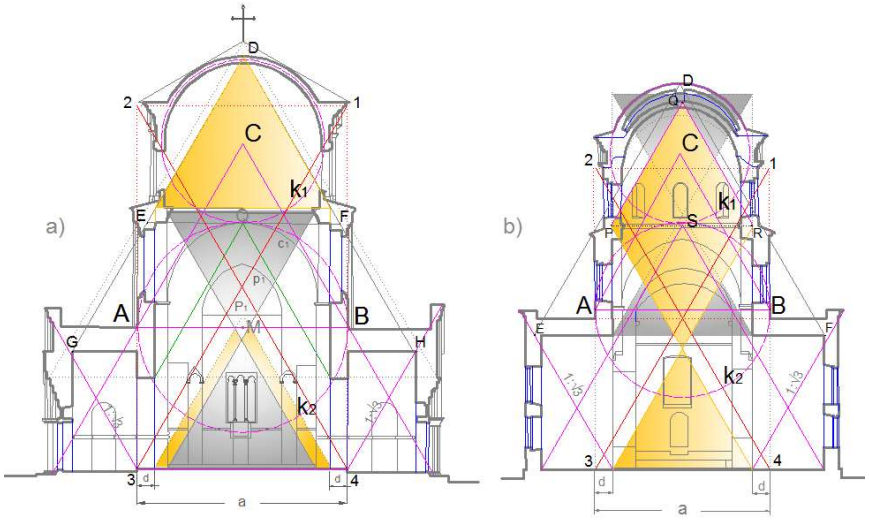

Fig. 8. *Mirroring geometry* of triangles: cross sections of Studenica – a) and Žiča - b)

2.3 Proportional analysis of "empty" spaces by classical constructions

Beside general triangular proportional geometric scheme, which is presented both by Štambuk's proportional canon and by complementary triangular scheme, the analyses of the architectural composition are additionally considered in more classical geometric system. Geometric regularities of the three particular types: proportional roots, ratio expressed by whole numbers and golden mean ratio are examined. The approximation of values is adopted regarding inaccurateness of builders' performance regarding the interior or "empty" space. Proportioning of narthexes, first and central bays show significant resemblance in both layouts. Although the concept of the altar space is due to traditional influences (three-partite altar in Studenica; separate sanctuary and side *pastophoria* in Žiča), proportional ratio 1:2 appears both in *pastophoria* and *diakonikon* /*prothesis* (Fig.9). Golden mean ratio (1:1.618)[1] appears in dimensioning of altar, central core and side chapels which is expected, according to the meaning of these spaces in liturgical sense. Geometric analysis overall results, both in layouts and sections are presented in Table 1.

[1] The most common approximated value for expressing golden ratio is 1.618.

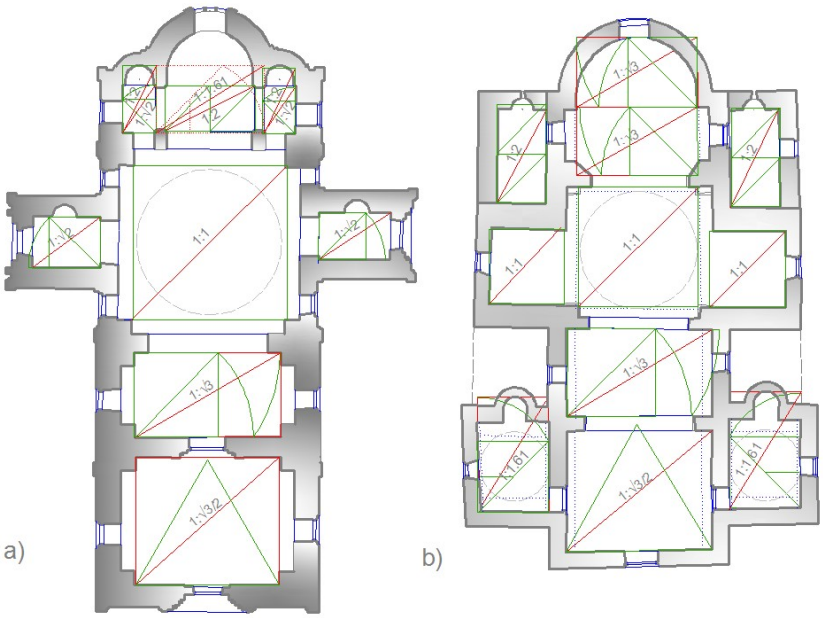

Fig. 9. Proportions of the interior spaces – layouts of Studenica –a) and Žiča-b)

Table 1. Two-dimensional proportional relations for selected interior - "empty" spaces

LOCATION	FUNCTION	ŽIČA		STUDENICA	
		Layout	Cross section	Layout	Cross section
Altar space	Sanctuary	1: √3	1: √3	1:2	1: √7
	Diakonikon/Prothesis	1:2	1: √2	1: √2	1: √30
Central space	Central core	1:1	1: 1.618	1:1	1: √2
	1st isle	1: √3/2	1: √3	1: √3	1: 1.618
	Vestibule/Choir	1:1	1: √3	1: √2	1: √7
Narthex	Narthex	1: √3/2	1: √3	1: √3/2	1: 1.618
	Side chapel	1:1.618	1:√5	-	-

3 CONCLUSION

Comparative analysis of proportions of two monumental Serbian medieval monastery churches guided by the two geometric means – regular geometric shapes and classical constructions of a ratio (proportional roots, golden mean, or numeric ratios), demonstrates the existence of regularities both in layouts and cross sections of the two selected structures. Characteristics of the so-called Raška style to which these two churches belong, are related to their architectural program and specifics of the struc-

tural parts of the church. Although majority of scholars who study Serbian art and architecture have already examined architectural composition, massing, structural elements, or medieval measuring units applied on diverse monuments belonging to specific style group, geometric analysis accompanied by drawings are rather rare. Hence, this current work contributes in terms of detailed geometric analysis, supported by point cloud data, and with adequate graphic representation.

The two churches in Studenica and Žiča monasteries, although differing in architectural program and building purposes, have several characteristics in common regarding their proportions:

- triangular geometric schemes are inscribed in layouts, where three equilateral triangles by *mirroring* geometry (along the longitudinal axis) “measure” the length of the church nave;
- triangular geometry when inscribed in cross sections “measures” the total height of these two churches: a single equilateral triangle is the measure of a dome structure (cupola and drum joined), while two triangles in vertical *mirroring* geometry "measure" supportive structure of the central cores;
- layout proportioning of the *narthex* (1:$\sqrt{3}/2$), 1st bay (1: $\sqrt{3}$), central core (1:1) and *pastophoria* (1:2) reveal the use of the classical constructions of a ratio;
- proportioning of the central core by golden mean points to the significance of the underlying structure of the dome [19]

The importance of this research lays in its practical results which can be verified or questioned on a larger sample of monuments. The other possible application of the results can be used for developing valuable proportional data for the restoration or reconstruction of ruined monuments [20,21]. Moreover, in larger historical context, such data widen our knowledge about medieval architects and their designs.

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