This is the copy of the accepted manuscript of the article published in

Nexus Network Journal

Architecture and Mathematics

ISSN 1590-5896 (online)

online version from 23 January 2019 is available at
https://doi.org/10.1007/s00004-018-00426-z

The final publication is available at link.springer.com

ISSN 1522-4600 (print)

Nexus Netw J (2019) 21:33-58

Geometric proportional schemas of Serbian medieval Raška churches based on Štambuk's proportional canon

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based on Štambuk's proportional canon

Abstract

In this paper, an analysis of three Serbian medieval Raška churches highlights the significance of the interaction of regular geometric shapes in the composition of their underlying proportional scheme. The complex geometry recognised in the layouts and cross-sections of these monastery churches points to the potential use of a unique triangular proportioning system. This proportional system is derived from the initial geometry of Štambuk's proportional canon, which employs two circles constrained by an equilateral triangle in a specific setting. As the essence of religious medieval structures is often related to their interior, in this paper 3D "empty space moulds" are created by the parametric modelling of each element of the structure. This research shows how the variation of the two critical points of the church interior, the centres of the dome and of the main apse, along with the key triangles incorporated in the proportional scheme, affect the final 3D structural model.

Keywords: Proportions, Geometric scheme, Orthodox church, Raška style, Parametric modelling.

Introduction

As one of the oldest theoretical concerns in architecture, proportion plays a crucial role in design. Recurrently, behind proportioning is a tendency for architects to try to achieve beauty, harmony and order in design. In his text "Two kinds of proportions," Cohen (2014) speaks of "proportion-as-ratio" and "proportion-as-beauty." He explains how the quantitative/qualitative ambiguity of the term *proportion* is related to an object. "Proportion-as-ratio" is an abstract quantitative comparison of objects' dimensions, whereas "proportion-as-beauty" is related to a qualitative aesthetic assessment. By embracing these two complex definitions, this paper analyses the proportioning of religious structures. The analysis is based on the geometry of regular shapes and their constraints, obtaining proportion-as-ratio measures for the objects' dimensions. Several issues are highlighted in this way:

- the use of a unique proportioning system based on dominant geometric shapes (circle, square, triangle, etc);
- complex geometric construction (scheme or canon) as a tool for proportioning;
- the adjustment of a geometric/proportional concept to the purpose of the structure (i.e. its religious content);
- the symbolic use of geometric shapes in proportioning;
- the interaction of a 2D proportional scheme with a 3D structure.

The unbreakable connection between an architectural design and its geometry was established by Vitruvius. In the first century BC, this Roman architect, civil and military engineer, and author of the treatise *De Architectura*, claimed that an educated architect should know both arithmetic and geometry (Frings 2002). Since then, geometry has represented a strong foundation for subsequent investigations of proportions in architecture. Various building traditions have offered a wide range of systems for proportioning buildings using ratios, regular geometric shapes, geometric constructions, or shapes that could be interpreted using whole numbers (e.g. right angle triangles 3:4:5). Additionally, among such rules are: (i) dynamic rectangles related to square roots (e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.); (ii) the golden section (ratio expressed by the equation a/b=(a+b)/a); and (iii) "continuous division", related to the Fibonacci sequence of numbers translated to rectangles (Kappraff 1996; Dabbour 2012; Letić 2012; Roldan 2012). All of these can be easily constructed geometrically by using a compass and ruler. Kappraff (1999) claimed that proportional systems based on ϕ[2], $\sqrt{2}$ and $\sqrt{3}$, used for the design of buildings in antiquity, are the simplest ones exhibiting the two properties of repetition and harmony, both necessary for achieving beauty. Although the theoretical background for such rules existed (Heath 1908), it is neither possible to confirm, nor to deny that their application was a result of a designer's intention, as written records have not been preserved.

[2] "The system of proportions based on ϕ where $\phi=(1+\sqrt{5})/2$."

In *De Architectura* Vitruvius suggests that proportioning is a guide for understanding Greco-Roman architectural practice (Suppes 1991). Indeed, some examples of proportioning based on regular geometric shapes are present in Roman architectural design of public structures (e.g. arenas, aqueducts, cisterns), where a square grid and compass were applied both in the ground plans and elevation patterns (Stewart 2015). *Ad triangulum* proportioning principles were also used in complex design schemas of medieval Gothic cathedrals (Bork 2014). In Byzantine religious architecture, the correlation of a square and circle is manifested in a three-dimensional manner through the development of a spherical dome over a cubed space of built edifices (Kurtović-Folić 1996). Religious structures, in relation to their specific purpose, additionally aim to point to transcendence as an essential frame, and may have involved a stronger symbolic use of geometry in proportioning than other public buildings. For example, Kurtović-Folić (1996) explains that the Byzantine tendency to create a church structure that represents a materialised version of the Cosmos was crucial for all aspects of constructing. Potamianos et al. (1995) studied the design of Byzantine churches and also highlighted the geometric adjustments of proportions of a church structure needed for its functional, liturgical, purpose. Guided by Kalligas' and Moutsopoulos' theories regarding the geometry of Byzantine churches, Potamianos and others have provided a sophisticated geometric analysis and parametric modelling that suggested a critical role in church design based on the visual perception of a faithful walk-through of the church's interior space towards the central "figure" – the Christ *Pantocrator* depicted at the apex of the central dome. The results of this approach connected the monumental decoration of the church interior and the overall architectural design. The inclusion of parametric modelling in this research allows for studies of various solutions for a church structure and promises a better understanding of historical architecture and its design principles. Further development of 3D parametric modelling strengthened by the advances in surveying and measuring techniques, initiated a new platform for the purposes of cultural heritage - Historical Building Information Modelling (HBIM).

The aim of this paper is to contribute to the studies of Serbian medieval sacred structures in the context of the proportioning and geometry present in their architectural design. Here, the proportional analysis is done for three medieval Orthodox churches of the so-called Raška style in Serbia. In particular, the authors examine the applicability of Štambuk's little-studied proportional canon (2002), through the proportional schemas composed of the geometric shapes of a circle and equilateral triangle. Štambuk's proportioning is based on the concept of the *holy* triangle, the key element of his proportional canon hypothesised to be relevant for medieval Christian structures.

By developing a better understanding of the geometric principles in these three churches, the paper supports further efficient reconstruction of ruined monuments with similar building complexity and within the same group. The research is also significant because it points to the potential interpretation of the interior church space and its parts, using a 3D parametric model. It also represents an introduction to the HBIM platform for Serbian cultural heritage monuments.

Related research on the proportioning of medieval Christian churches

The three medieval Orthodox Christian churches in the territory of the former Raška state (current Serbia), which are the subject of the current research, share a unique architectural style, usually interpreted as being built under the dominant influences of both the Byzantine and Romanesque traditions (Čanak-Medić 2007; Ćurčić 2010). Several scholarly approaches to their proportional analysis, using applied geometric shapes, measuring units and the symbolic and semantic nature of the church space may be considered.

The particular architectural style of Serbian medieval churches was often influenced by available material resources and building techniques, which affected the accuracy of their construction. Awareness of this issue is present in the research of local scholars and experts in medieval studies. These materials offer analyses of numerous churches' measurements where even within the same building, several different measurement units can be identified (Bošković 1956-1957; Čanak-Medić 1986; Nenadović 2003). This fact has been variously explained as either a result of the levelling of the terrain and other deformations that occur over time, or inaccuracies resulting from construction method and applied building materials. For example, Nenadović (2003) claims that the churches built with marble have shown less deviations from orthogonality than those made of rough stone. The measuring units of medieval Raška ("foot", "elbow"[3], "inch", etc) were incorporated into *moduli* and recognised within three geometric systems: *ad quadratum*, *ad triangulum* and the *golden division* (Čanak-Medić 1986; Čanak-Medić 1995; Pejić 2002). Čanak-Medić (1986), in her study of the Studenica monastery church, recognised *ad quadratum* principles of proportioning in the layout, enriched with a square modular grid where one *module* is equal to a triple measure of the unit "foot" (approx. 31.5 cm according to Čanak-Medić). Corcuff (2012) recognised this approach, where the modular system leads to a grid, as a useful tool for architects.

Master builders recurrently used proportional compasses when establishing a geometric scheme for an object; one such compass with two pairs of jointed legs was found at the Gradac site during archaeological excavations (Čanak-Medić 1996). This fact suggests the intention of the architect to obtain a harmonic relationship between the elements of an architectural composition. Additionally, proportional analysis of the sixth century churches at the Byzantine site of *Iustiniana Prima* reveals the multiple use of the square as a primary shape for church layout proportioning, which resulted in proportional ratios 3:5 and 4:7 of the object's width/length. These proportions are in contrast to those based on Pythagoras' triangle (3:4:5 ratio), which are also present at this particular site (Petrović 1956-57).

Searching for a deeper understanding of design principles of religious objects, Štambuk (2002) proposed that the geometric scheme of churches can be related to the altar table, the apex of the dome[4], and the top point of the apse, all of which highlight the symbolic and semantic role of the architectural space. He pointed to the pervasive use of the circle and equilateral triangle in the proportional systems of numerous churches in the wider Mediterranean region and across a large chronological span from the fourth to the late fifteenth centuries (Štambuk 2002, 2014 and 2017). Štambuk recognised the use of this system in early Christian and early Byzantine churches (fifth and sixth centuries) in the Mediterranean region, including less known religious structures of the Christian community in Zurzach (Tenedo), modern Switzerland, the church of Theotokos on Mt. Gerizim, as well as the famous Justinian's foundations of St. Vitale in Ravenna and SS. Sergius and Bachos and Hagia Sophia, in Constantinople (modern Istanbul). He also tested his system on several smaller and later churches on the island of Hvar in modern Croatia.

Proportional analysis of the three Serbian medieval churches, based on Štambuk's theory

The main churches of the monasteries Studenica, Žiča and Gradac, representing the Raška style (twelfth to fourteenth centuries), were selected from a wide range of Serbian monuments, because of their quality, complexity and overall importance (Nenadović 1987). The Studenica monastery is on the UNESCO World Heritage Sites list (1986), while the other two churches are under the protection of the National Institute for Protection of Serbian Cultural Monuments, and are categorised as heritage of exceptional significance (Bogdanović 2015). The selected churches were built by three generations of patrons from the Serbian Nemanjić dynasty. Studenica monastery was Stefan Nemanja's major legacy, Žiča was the coronation church of his son Stefan, while the patron of the last one, Gradac, was Queen Helen of Anjou, the wife of King Uroš, Nemanja's grandson.

Despite their age, these medieval buildings are sufficiently well preserved to allow for proportional analysis that may point to the original guiding principles behind their design. The precise technical documentation of the main churches of the monasteries of Studenica, Žiča, and Gradac are derived from Čanak-Medić (1986; 1995) and Kandić (2005) and are used for the proportional analysis in the current study. Additionally, existing documentation was complemented with terrestrial laser scanning data on the Church of the Mother of God in Studenica, carried out by the authors.

The key geometry for proportioning in the work of I. Štambuk

Štambuk's proportioning (2002; 2014; 2017) relies on the use of a circle and equilateral triangle. The strict geometric constraints of the two circles and an equilateral triangle, applied to the layout of the church of St. Vid, located on the island of Hvar, represent Štambuk's proportional canon (Fig. 1a). The main equilateral triangle, which Štambuk calls the *holy* triangle (pink triangle), is related to the church nave's width (exterior) while its vertex coincides with the centre of the apse. This signifies the most sacred place of the church – the position of the altar table in the sanctuary. The smaller circle $\boldsymbol{k_2}$ overlaps the exterior edge of the *apse* and it has a common tangent with the bigger circle $\boldsymbol{k_1}$. Moreover, the edge of the *holy* triangle coincides with the diameter of $\boldsymbol{k_1}$. The other regularity appears in the setting of a smaller triangle, whose vertex coincides with the interior top point of the *apse,* while the opposite edge is tangential to both circles. The tangent line in

[4] The common "points" where the icon of Christ is painted in the church interior are the apex of the dome and the top point of the apse (interior altar).

Štambuk's description represents the boundary of the sacred space; it separates the church sanctuary with the altar from the public space of the church nave. In his analysis of the use of geometry as a guiding principle for proportioning and church design, Štambuk emphasises the ratio 1:√3 as a proportion of the boundary (church layout) rectangle: width of the rectangle **- $2r_1$** / length of the rectangle - **$2r_1$+$2r_2$** (Fig. 1b).

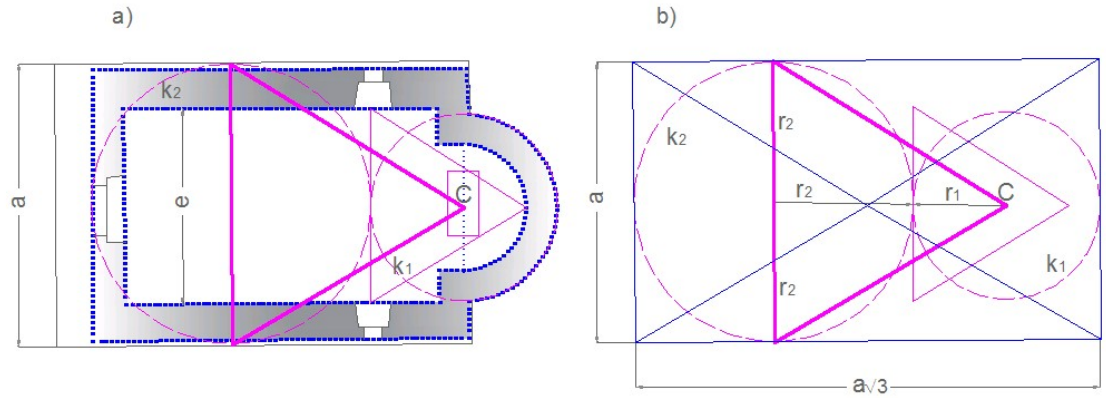

Fig. 1 Proportional canon by I. Štambuk: a) Layout of St.Vid church in Velo Grablje, Croatia; b) Pure proportional geometric scheme of the church's architectural design; Dragović after Štambuk (2002).

In the present study, this proportional key geometry is, for the first time, examined in the churches of the *Raška* group. The relevance of this analysis is additionally supported by the already observed resemblance of the architectural concepts in the layouts of the twelfth century Benedictine church of St. Mary on the island of Mljet (in the territory of Croatia), usually associated with the Romanesque style, and the almost contemporaneous church of the Mother of God in the Studenica monastery (Tomas 2011).

Proportional Schemas of the Three Churches in the Studenica, Žiča and Gradac Monasteries

In order to compare the principles used in the design and dimensioning of the layouts/sections of the three churches in Studenica, Žiča and Gradac, the proportional analyses of their exterior dimensions are presented in Table 1, where **a** is nave width; **b** is total length; **s** is total width (cross-section); **n** is length without narthex (*unit is metre).

Table 1 Proportional ratio of characteristic dimensions of the three churches

Monastery Churches	**Dimensions***				**Ratio**			
Location	a	b	s	n	s/a	s/b	a/b	a/n
Žiča	8.22	24.83	14.64	18.04	1: 1.78	1:1.69	1:3.02	1:2.19
Studenica	10.11	27.27	17.94	20.31	1: 1.77	1:1.52	1:2.69	1:2
Gradac	8.75	25.48	14.26	19.42	1: 1.63	1:1.78	1:2.91	1:2.22

Triangular proportional scheme of the church in Studenica

The proportional analysis of the church in the Studenica monastery shows the existence of the two characteristic equilateral triangles (blue and red triangles), both in the floor plan and the cross-section (through the dome). They are also observable in the overall structural composition. The characteristic key points for the positioning of these triangles are related to the elements of Štambuk's construction: the two circles and the *holy* triangle (Fig.2a-b). Moreover, the strict relationship of the circle k_1, the equilateral triangle – the *holy* triangle (magenta coloured highlighted line) – and the circle k_2 exists. This confirms the applicability of Štambuk's *proportional canon* regarding the determination of the sacred space of the altar and central core, along with the boundary line between them, both in the floor plan and cross-section respectively. Additionally, the circumscribed rectangle of the church nave (without apses and narthex), that of the ratio 1:√3, is constrained to the key point – the centre point of the main apse.

The correlations between the characteristic elements (the vertex, edge, angle, etc.) of each analysed triangle (red, blue, *holy* and small triangles) and the elements of the church structure as well as the wall contours are presented in Table 2.

Table 2 Geometric correlations between the characteristic triangles, circles, structural elements and the whole structure of the church in the Studenica monastery

Shape	Geometric element	Correlations in the **layout**	Correlations in the **cross-section**
red triangle	vertex	***M*** – entrance wall's midpoint	***S*** – centre of the dome D1
	edge – ***s***	total exterior width	total exterior width
	height – h_S	vestibule's west walls position	elevation of ***S***
blue triangle	vertex	***P*** – top point of the apse A1	***E*** – exterior top point of D1
	edge – ***e***	total interior width	total interior width
	height – h_E	arch L3 / vertex P	vault V2 / top of the dome D1
	angle	**δ** – apse A1 openings	——
holy triangle	vertex	***S*** – centre of dome D1	***C*** – centre of the apse A1
	edge	**c** – central core width	**a** – exterior nave width
	angle	γ – arch L6 openings	——
	circle – k_1	exterior apse curve	axis of dome D1
	circle – k_2	diameter of k_2 = **c**	diameter of k_2 = **a**
	tangent line k_1 / k_2	axis of the arch L6	elevation of the arch L3
small triangle	vertex	***P*** – top point of the apse A1	interior top point of the dome D1
	edge	*iconostasis* width	interior dome radius

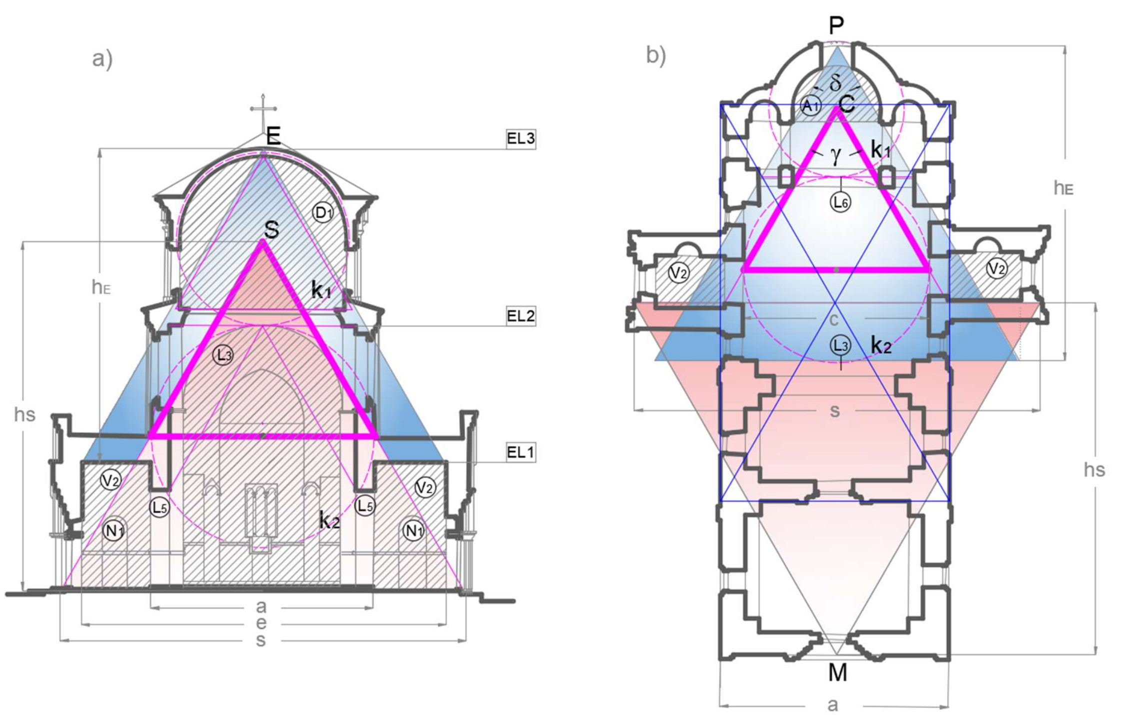

Fig. 2 Triangular geometry of the proportional scheme of the church in Studenica; a) cross section, b) floor plan. Dragović after Čanak-Medić (1986)

Furthermore, the hierarchy of the three key triangles in the cross-section (red, blue and holy triangles) and their geometric constraints, mostly determined by Štambuk's construction, is in compliance with the three structural zones of the church (Bogdanović 2017): vestibule vaults – zone I (EL1), central supporting arches – zone II (EL2) and dome/drum structure – zone III (EL3).

Proportional scheme of the church in Žiča

The expanded architectural programme of the church in the Žiča monastery is related to a wider altar space with the large apse and separate pastophoria, the open choir space and side chapels connected to the narthex, as well as to the specific purpose of the church and the coronation ceremony of Stefan Nemanjić. Hence, the geometric design is here more complex and rich. The lack of orthogonality of the walls, critical in the church layout and here probably as a result of less skilled builders, resulted in deviations from the regular geometry of the triangles when applied in the geometric scheme. However, several key points and triangles, present in Studenica's scheme, which may be considered the base-prototype pattern, are used in Žiča for its more complex geometric and structural composition. The two additional triangles appear in the layout in accordance with the insertion of the side chapels (Fig.3a-b). The smaller inscribed triangle more clearly appears in the interior of the altar space, as well as inside the dome structure, while defining the size and contours of the spatial elements. The base edge is the line tangential to the two circles $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$.

Table 3 Geometric correlations between characteristic triangles, circles, structural elements and the whole structure of the church in the Žiča monastery.

Shape	Geometric element	Correlations in the **layout**	Correlations in the **cross-section**
red triangle	vertex	***M*** – entrance door midpoint	***T***– midpoint of drum's base
	edge – ***e***	total interior width	total interior width
	height – $\boldsymbol{h_T}$	the position of the vaults V2	elevation of the dome structure
blue triangle	vertex	***P*** – top point of the apse A1	***E*** – interior top point of D1
	edge – ***e***	total interior width	total interior width
	height	$\boldsymbol{h_P}$ – arch L6 / vertex P	$\boldsymbol{h_E}$ – vault V2/top of the dome D1(E)
holy triangle	vertex	**C** – centre of the apse curve	***S*** –centre of the dome curve
	edge	**a** – exterior nave width	**a** – exterior nave width
	angle	γ – arch L7 openings	–––
	circle – $\boldsymbol{k_1}$	offset curve (apse exterior)	dome D1 exterior curve
	circle – $\boldsymbol{k_2}$	diameter of $\boldsymbol{k_2}$ = **a**	diameter of $\boldsymbol{k_2}$ = **a**
	tangent line $\boldsymbol{k_1 / k_2}$	altars' inner wall edge	elevation of the drum
small triangle	vertex	interior top point of the apse A1	interior top point of D1
	edge	interior altar width	***c*** - central core width
additional triangles	vertex & edge	***C*** – centre of the apse A1 ***t*** – total exterior width (chapels)	–––
		R – midpoint of east choir walls ***r*** – total interior width (chapels)	–––

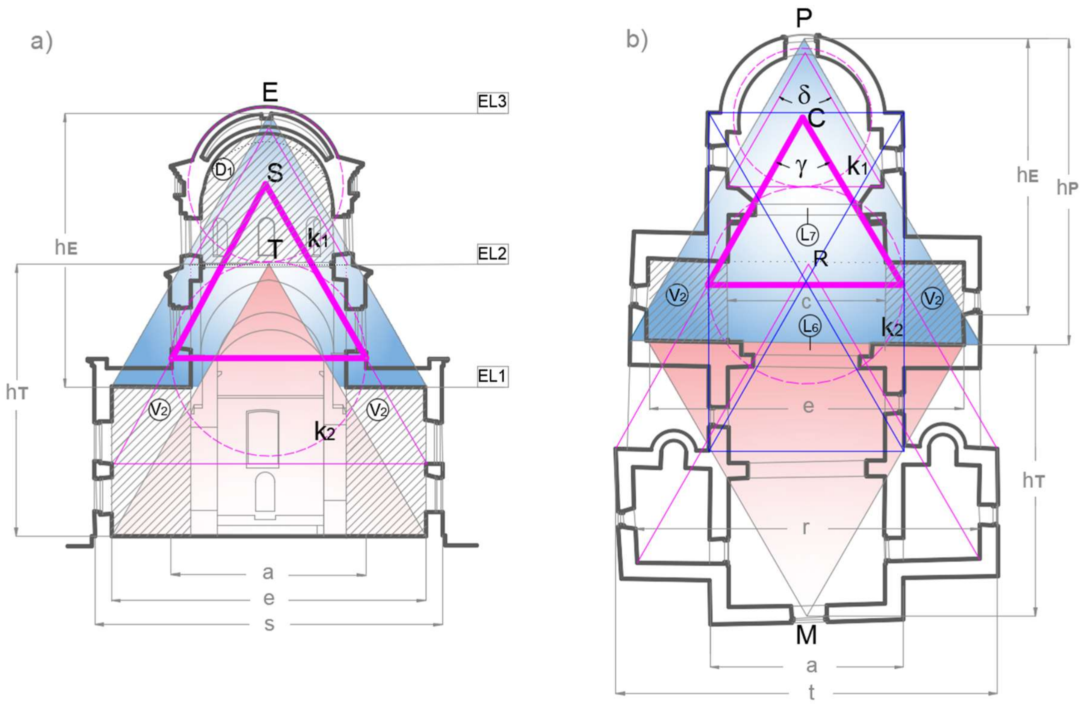

Fig. 3 Triangular geometry of the proportional scheme of the church in the Žiča monastery; a) cross section, b) floor plan. Dragović after Čanak-Medić (1995)

Proportional Scheme of the Church in Gradac

Although significantly irregular in terms of its construction (Nenadović 2003), the church in the Gradac monastery retains the main principles of triangular geometric design, previously shown in the cases of Studenica and Žiča. Its architectural programme is a combination of the two previous cases: the central core space is opened towards the choirs, the side chapels are connected to the narthex (Žiča) and the tripartite altar space (Studenica). The geometric scheme of the cross-section is closer to that used in Studenica, while its tripartite altar design (visible in the layout) is more elongated. When applied to the church layout, Štambuk's canon, specifically its characteristic tangent line, has lost the attribute of a sacred and public spaces boundary. Moreover, the analysis presented here reveals two new key points – the vertexes of the additional equilateral triangles in the triangular proportioning scheme.

Table 4 Geometric correlations between characteristic triangles, circles, structural elements and the whole structure of the church in the Gradac monastery.

Shape	Geometric element	Correlations in the **layout**	Correlations in the **cross-section**
red triangle	vertex	***M*** – entrance door midpoint	***S*** – centre of the dome D1
	edge – ***s***	total exterior width	total exterior width
	height – h_S	vaults V2/vertex M	centre of the dome D1
blue triangle	vertex	***P*** – top point of the apse A1	***E*** – exterior top point of D1
	edge – ***e***	total interior width	total interior width
	height – h_E	***PE'*** length (E' – projection of E)	vault V2 / top of the dome D1(E)
holy triangle	vertex	***C*** – centre of A1 exterior curve	***S*** – centre of the dome
	edge	***c*** – interior nave width	**a** – exterior nave width
	angle	γ – arches L2 openings	—–

	circle – $\boldsymbol{k_1}$	apse A1 exterior curve	dome D1 interior curve
	circle – $\boldsymbol{k_2}$	diameter of $\boldsymbol{k_2}$ = **c**	diameter of $\boldsymbol{k_2}$ = **a**
	tangent line $\boldsymbol{k_1/k_2}$	*deviates from principles	——
small triangle	vertex	interior top point of the apse A1	interior top point of the dome D1
	edge	interior altar width	***c*** - central core width
additional triangles	vertex	***Q*** – meeting point of $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$ ***E'*** – projection of dome centre	——
	edge – ***u***	total exterior width (chapels)	——

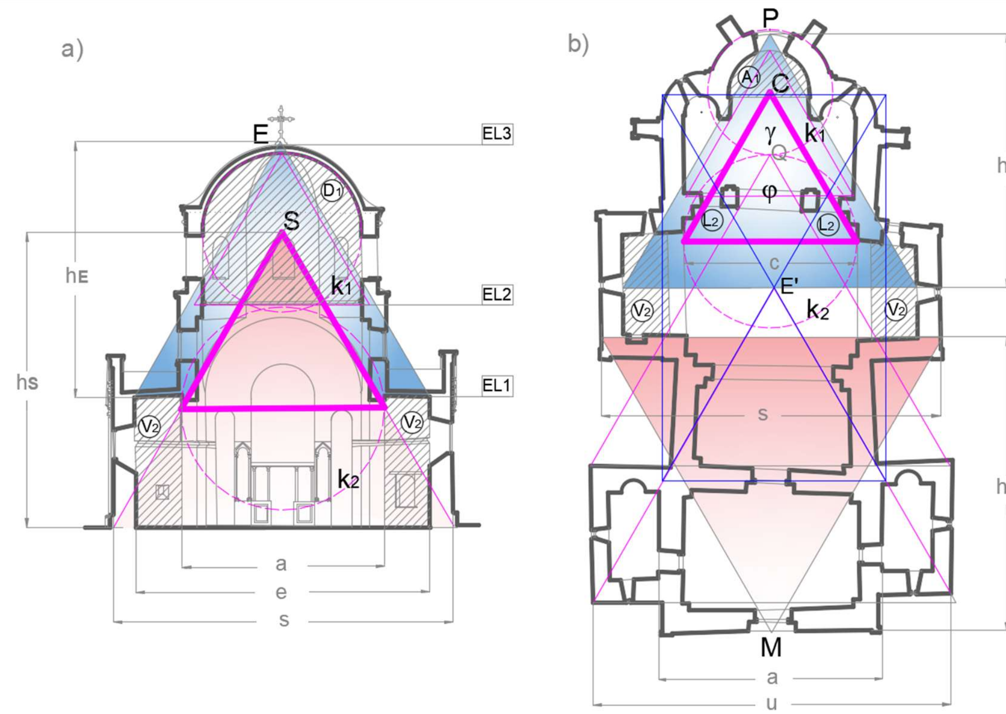

Fig. 4 Triangular geometry of the proportional scheme of the church in the Gradac monastery; a) cross section, b) floor plan. Dragović after Kandić (2005)

Pure geometric proportioning schemas

The analysis of each of the three churches reveals regular geometric schemas, representing the underlying geometry of their design. For each of the three patterns only two variables, beside wall width, are required for the scheme construction: nave width-***a*** (width of the key rectangle), and total width-*s* (***e***). The graphic representations of the three patterns are presented in Figs. 5-6.

The geometric construction procedure of the layout of pattern "S" starts by drawing the red equilateral triangle (given edge – ***s***) and blue rectangle, in a ratio of 1:√3 (given edge - ***a*** of equilateral triangle) in such a constraint whereby the crossing of the rectangle's diagonals coincides with midpoint of edge ***s*** of the red triangle. A new triangle is inscribed in the upper half of the blue rectangle, defining point ***C***-vertex of the *holy* triangle. The holy triangle's edge dimension is ***a-2d,*** where ***d*** - wall width. If the two circles $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$ are drawn following the constraints of Štambuk's canon, the blue triangle with vertex ***P*** at the apex of $\boldsymbol{k_1}$ and the opposite edge tangential to $\boldsymbol{k_2}$, close the graphic procedure (Fig. 5). The other two schemas (Fig. 5b-c) rely on the same constraints of the rectangle (1:√3) and the two key triangles (red and blue), while the two additional triangles are constrained to characteristic intersection points (crossing of the diagonals of the rectangle, point C, or the meeting point of the circles $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$).

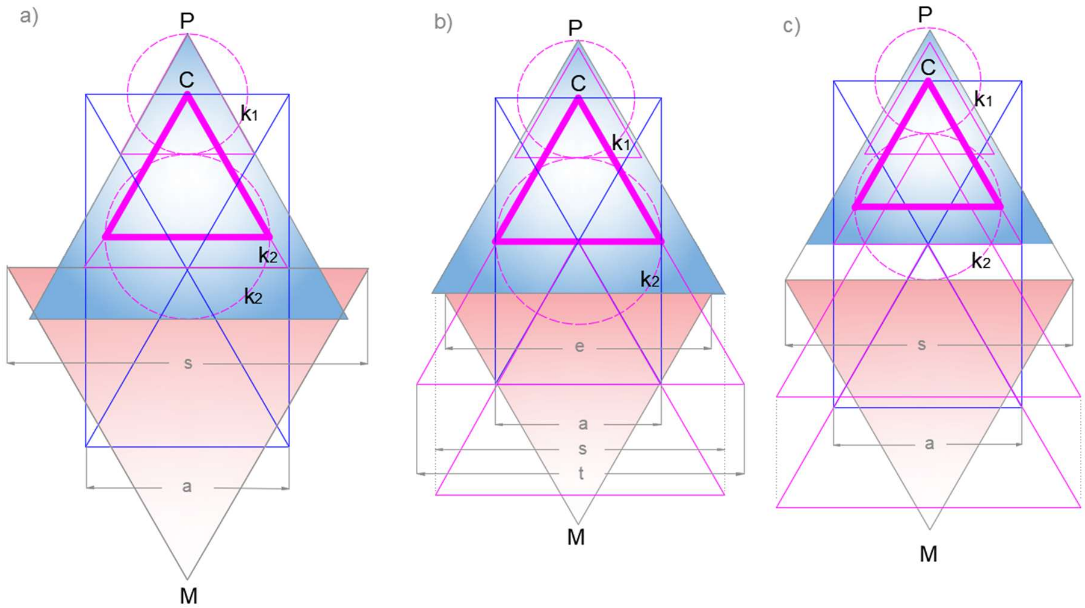

Fig. 5 Three geometric proportional schemas: variations of church layouts; a) pattern "S", b) pattern "Z" and c) pattern "G".

The corresponding geometric patterns were created for the cross sections of the three churches. The construction procedure of the pattern "S" starts with the setting of the red triangle (given edge ***s***). The top point ***S*** of the triangle is the key point, i.e. the centre of the dome. The *holy* triangle is set according to the given edge ***a.*** After constructing the circle $\boldsymbol{k_2}$ (radius of $\boldsymbol{k_2}$ is equal to ***a/2***), and circle $\boldsymbol{k_1}$, the point ***E*** **-** vertex of the blue triangle is defined. The edge dimension of the blue triangle is of the same size as the corresponding one in the layout scheme (Fig.6a). The two construction procedures of patterns "Z" and "G" start with the setting of Štambuk's elements, the *holy* triangle (given edge ***a***) and the circles $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$. The further construction flow of the red triangle depends on the given edges ***e*** and ***s*** and their top points ***T*** and ***S*** respectively.

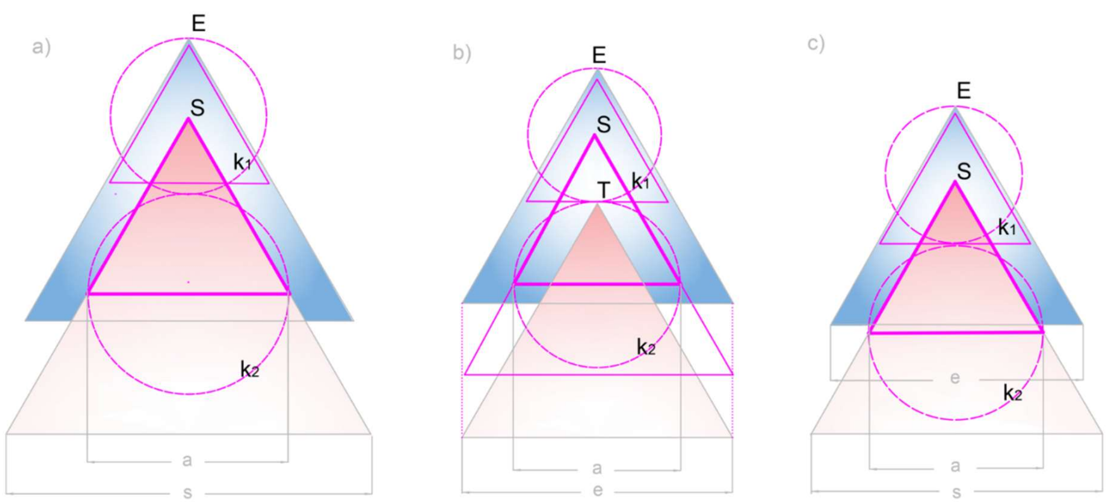

Fig. 6 Three geometric proportional schemas: variations of church cross sections; a) pattern "S", b) pattern "Z" and c) pattern "G".

The ratio of the given dimensions **a**/**s** obtained in the three schemas is in accordance with the values from Table 1: a/s=1:1.78 (pattern "S"); a/s=1:1.77 (pattern "Z"); a/s= 1:1.63 (pattern "G"). Any variation of the ratio ***a***/***s*** (***e/s***) in particular geometric schemas would need expert skills, because the geometric constraints between the elements of the scheme would change and consequently affect the appearance of the structural model.

Parametric Modelling of the Interior Church Structure

The 3D models of the church interiors in Studenica, Žiča and Gradac are critical for recent standardised approaches in the architectural practice of creating models based on a parametric library of structural elements (Murphy 2011; Čučaković 2016; Banfi 2016; Quattrini 2017; Lopez 2018). In the context of the creation of a parametric library (cf. Murphy 2011), the semantic partition of 3D modelling required the creation of a repository of samples of parametric "inverse" 3D structural models based on families of elements characteristic for church architectural design (cf. Quattrini 2016). These elements are sorted into categories based on a variety of shapes and functions (arches, vaults, pendentives, apses, drums and domes). The elements are classified regarding their properties: geometry, dimensions, algebraic ratio, and location within the church structure (Tables 5-7). For each element, a graphic representation and annotation of its profile shape is provided and, in addition, the geometric construction of *dynamic*/*golden* rectangles for proportioning is applied (Fig. 7). In order to assess the accuracy of graphic constructions representing the ratios of dynamic rectangles such as $1:\sqrt{2}$, 1: 1.61 (the golden mean), $1:\sqrt{3}$, $1:\sqrt{5}$, $1:\sqrt{7}$, etc, a statistical analysis of the errors is performed. In order to determine the correlation between the ratio and its graphic approximation (columns 6 and 7 in the Tables 5-7) a Sperman's rank-order correlation is employed. According to the results, a strong positive correlation between the ratio and its graphic approximation is statistically significant ($\rho=0.982$; $p<0.001$) for each of the three monastery churches[5].

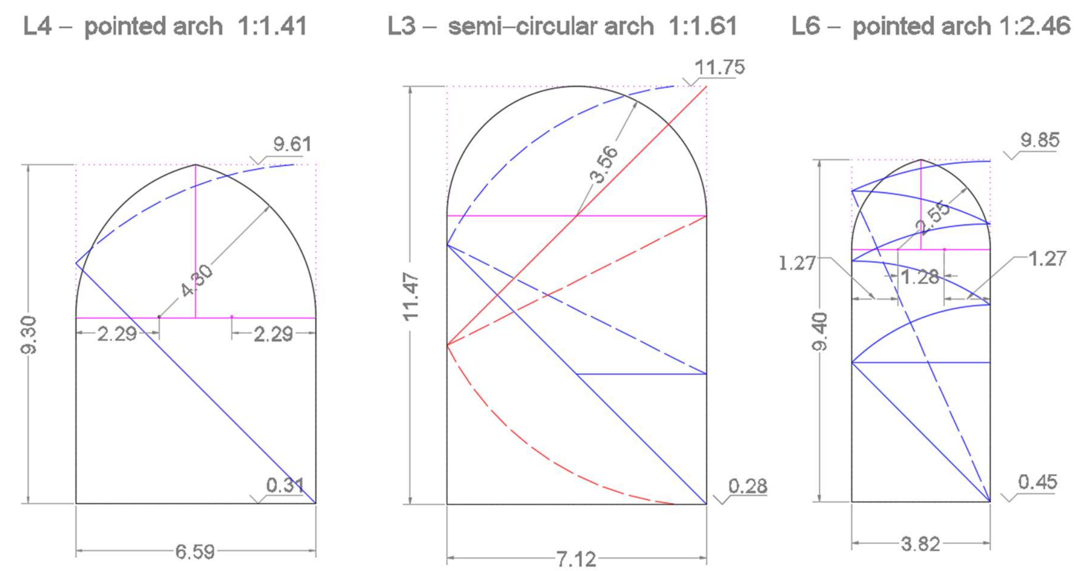

Fig. 7 Graphic representation of profile shapes with annotation; Geometric construction of dynamic rectangles; Elements of specification (Table 5): the arches from the Studenica monastery church.

Each element is modelled according to its spatial geometry and profile shape and parameterised (e.g. width, height, length, radius, etc.). The number of parameters is 2 to 4, depending on the complexity of the spatial forms (e.g. the geometry of the pendentives required 3 parameters for its shape definition), and their values are varied for each position of the element in particular specifications (Figs. 8-10).

The final 3D models of the interior spaces of the three churches were built by inserting mass models of their structural elements into the architectural project, where the layout drawing was set as a base for the structure. Autodesk – *Revit*, as the most relevant software based on the BIM platform, was employed for the modelling of library parts (elements of the structure), partially due to its advantages in model transformation (masses into structural elements), efficiency in geometric performance, and compliance with point-cloud data, among other processing possibilities. Specifically, the efficient insertion of library parts, while adjusting their proportions to the geometry of the layouts, enabled quick modelling of the whole structure.

[5] The details are available as supplementary materials of the manuscript.

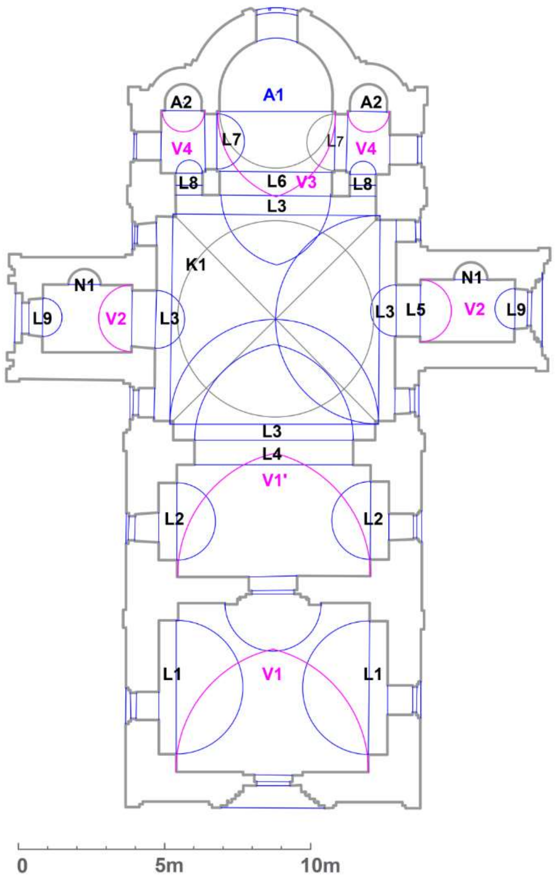

Fig. 8 Specification of structural elements: layout of the church in the Studenica monastery

Table 5 Specification of structural elements – the church of the Studenica monastery

OBJ.	No	Location	Dimensions w/h (cm)	Shape	Ratio	Graphic approximation
ARCHES	L1	narthex	451/655	semi-circular	1:1.45	$1:\sqrt{2}$
	L2	narthex	263/631	semi-circular	1: 2.39	$1:\sqrt{6}$
	L3	central core	712/1147	semi-circular	1:1.61	golden mean
	L4	narthex/core	659/930	pointed arch	1:1.41	$1:\sqrt{2}$
	L5	vestibule/core	186/423	semi-circular	1:2.27	$1:\sqrt{5}$
	L6	altar/core	382/940	pointed arch	1:2.46	$1:\sqrt{6}$
	L7	altar	190/490	semi-circular	1:2.57	-
	L8	altar/diaconicon	91/584	semi-circular	1:6.41	-
VAULTS	V1	narthex & 1st bay	659/1066	pointed	1:1.61	golden mean
	V2	vestibule	213/575	semi-circular	1:2.62	$1:\sqrt{7}$
	V3	altar /apse	401/1037	pointed	1:2.58	$1:\sqrt{7}$
	V4	pastophoria	145/793	semi-circular	1:5.47	-
APSE/ NICHE	A1	main apse	R 193/940	semi-calotte	1:2.45	$1:\sqrt{6}$
	A2	niche-altar	R 61/563	semi-calotte	1:4.61	-
	N1	niche-vestibule	R 57.5/335	semi-calotte	1:2.91	-

DRUM / DOME	D1	central core	R 366/311	cylinder	1:0.42	-
	K1	central core	R 366	semi-calotte	1:0.5	2:1

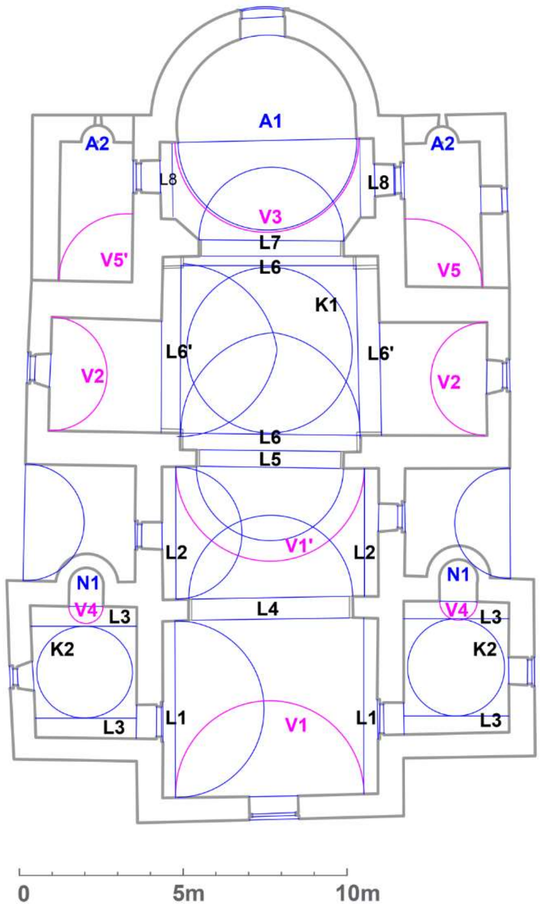

Fig. 9 Specification of structural elements: layout of the church in the Žiča monastery

Table 6 Specification of structural elements – the church of the Žiča monastery

OBJ.	No	Location	Dimensions w/h (cm)	Shape	Ratio	Graphic approximation
ARCHES	**L1**	narthex	534/695	semi-circular	1:1.3	-
	L2	1 st aisle	400/698	semi-circular	1: 1.74	$1{:}\sqrt{3}$
	L3	chapel	316/707	semi-circular	1:2.23	$1{:}\sqrt{5}$
	L4	narthex/ 1st bay	502/984	semi-circular	1:2	$1{:}\sqrt{4}$
	L5	1st bay /central core	449/970	semi-circular	1:2.16	-
	L6	central core	560/1075	pointed	1:1.92	-
	L6’	central core	525/1075	pointed	1: 2	$1{:}\sqrt{4}$
	L7	central core/altar	442/936	semi-circular	1:2.11	-
	L8	pastophoria	241/660	semi-circular	1:2.73	-
VAULTS	**V1**	narthex &1st bay	580/1021	barrel	1:1.76	$1{:}\sqrt{3}$
	V2	choir	345/630	barrel	1:1.82	-

	V3	altar	566/995	barrel	1:1.75	$1:\sqrt{3}$
	V4	chapel	108/422	barrel	1:3.91	
	V5	pastophoria	249/338	semi-barrel	1: 1.36	$\sim 1:\sqrt{2}$
	V5'	pastophoria	249/338	quarter-circular	1:1.36	$\sim 1:\sqrt{2}$
APSE/ NICHE	**A1**	altar	R 275/872	semi-calotte	1:1.57	~golden mean
	A2	pastophoria	R50/260	semi-calotte	1:2.60	
	N1	chapel	108/422	semi-calotte	1:3.91	
DRUM / DOME	**D1**	central core	R264/262	cylinder	1:0.5	2:1
	D2	central core	R149/185	cylinder	1:0.62	golden mean
	K1	central core	D264	semi-calotte	1:0.5	2:1
	K1	central core	D149	semi-calotte	1:0.5	2:1

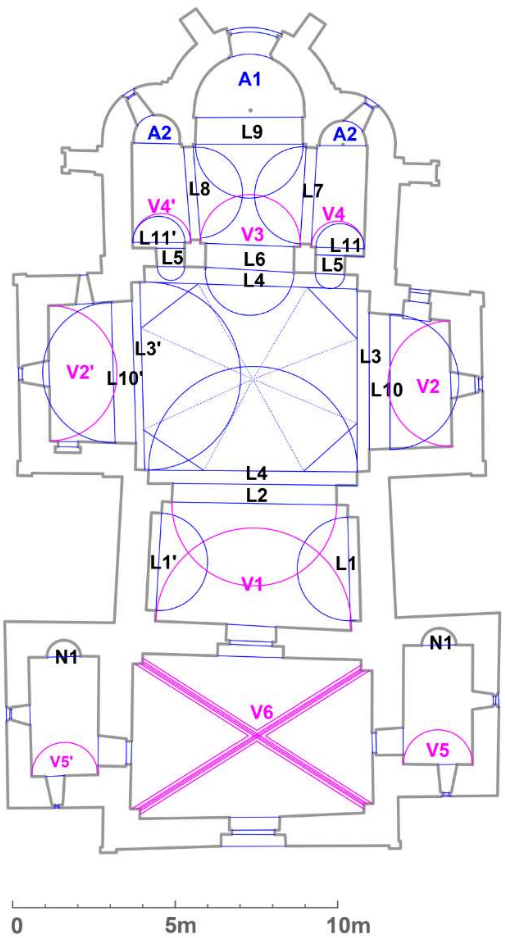

Fig. 10 Specification of structural elements: layout of the church in the Gradac monastery

Table 7 Specification of structural elements – the church of the Gradac monastery

OBJ.	No	Location	Dimensions w/h (cm)	Shape	Ratio	Graphic approximation
ARCH ES	**L1**	1st bay	313/645	semi-circular	1:2.06	$1:\sqrt{4}$
	L2	1st bay/central core	498/695	semi-circular	1: 1.39	$1:\sqrt{2}$

	L3	central core	557/877	semi-circular	1:1.57	golden mean
	L4	central core	642/889	semi-circular	1:1.38	$1:\sqrt{2}$
	L5	central core/ diaconicon	84/500	semi-circular	1: 5.95	-
	L6	altar/core	271/690	pointed arch	1: 2.55	-
	L7	altar/diaconicon	299/534	semi-circular	1: 1.78	$\sim 1:\sqrt{3}$
	L8	altar/prothesis	287/534	semi-circular	1: 1.86	-
	L9	altar apse	330/747	semi-circular	1: 2.26	$1:\sqrt{5}$
	L10	central core/choir	439/537	semi-circular	1: 1.22	-
	L11	pastophoria	149/589	semi-circular	1: 3.9	-
VAULTS	**V1**	naos-1st bay	596/937	barrel	1: 1.66	golden mean
	V2	central core/choir	386/557 422/562	barrel	1: 1.44 1: 1.33	$1:\sqrt{2}$
	V3	altar	294/901 347/901	barrel	1: 3 1: 2.6	$1:\sqrt{9}$
	V4	apse/pastophoria	163.3/710 155.3/710	barrel	1: 4.34 1: 4.57	-
	V5	chapel	216/402.5 213/402.5	barrel	1: 1.86 1: 1.89	-
APSE/ NICHE	**A1**	main apse/altar	R168.8/710	barrel	1: 2.1	-
	A2	small apses /altar	R72.55/470	barrel	1: 3.24	-
	N1	niches	R53/233	barrel	1: 2.2	$1:\sqrt{5}$
DRUM / DOME	**D1**	central core	665/394	octagonal prism	1:0.59	-
	K1	central core	665/298	segmented dome	1:0.45	-

Various churches of the *Raška* group may differ due to their diverse architectural programmes and proportions, while representing the structural order and creativity of their designers-architects and the skills of the master builders (Bogdanović 2011). The regular geometry of the parametrically modelled structural elements of the interior spaces in Studenica and Žiča has shown satisfactory adjustment to the layouts and cross-sections. In the case of the church in Gradac, the majority of the non-regular structural elements required ordinary instance modelling, due to their significant deviations from orthogonality and symmetry (Figs. 11a-c). Parametric modelling, in this case, does not make sense. Even if the geometry of some particular elements of the 3D models seem simple, the results of the proportional analysis and the geometric constraints of the structural elements confirm the complexity of the 3D models and make them valuable in the spatial reconstruction of the ruined churches (Dragović 2015).

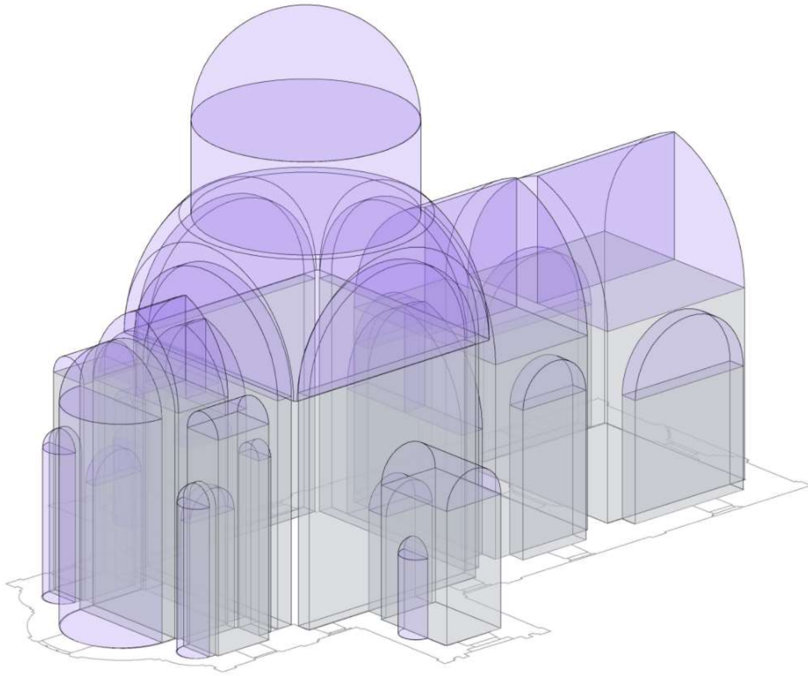

Fig. 11a Interior space model of the church in Studenica

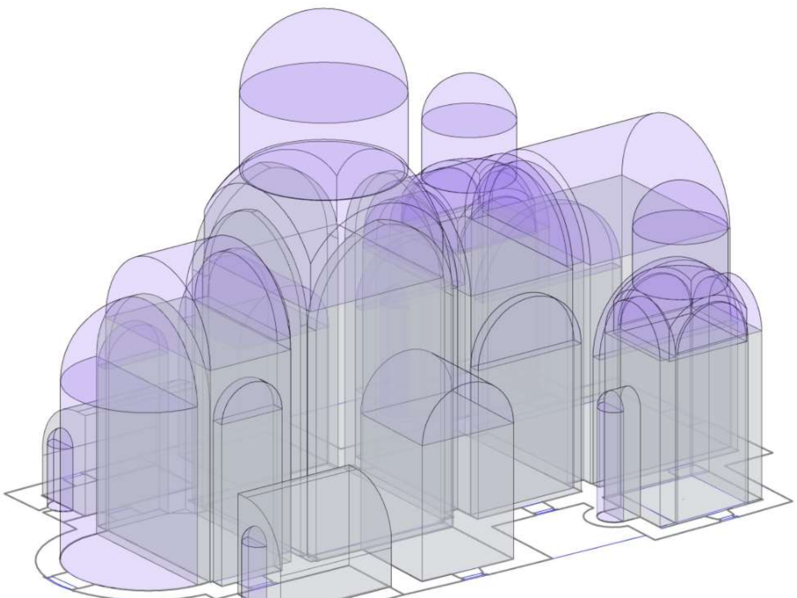

Fig. 11b Interior space model of the church in Žiča

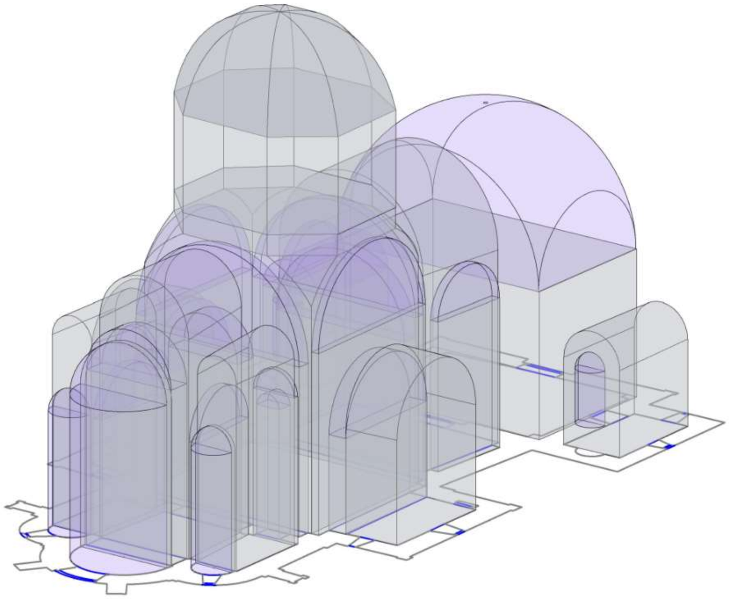

Fig. 11c Interior space model of the church in Gradac

After the modelling of the “empty moulds” of the interior church space, an assessment of the 3D model’s accuracy is obtained by the integration of the proportional scheme and 3D structural model of the church. The results of this integration are presented for the Studenica monastery church (Fig. 12a-b). It is obvious that some deviations of the model came from the geometric irregularity of the church layout. An *Autodesk Revit* 3D model, composed of mass elements, enables visual transparency of modelled elements. Accordingly, it enables ease of tracing errors.

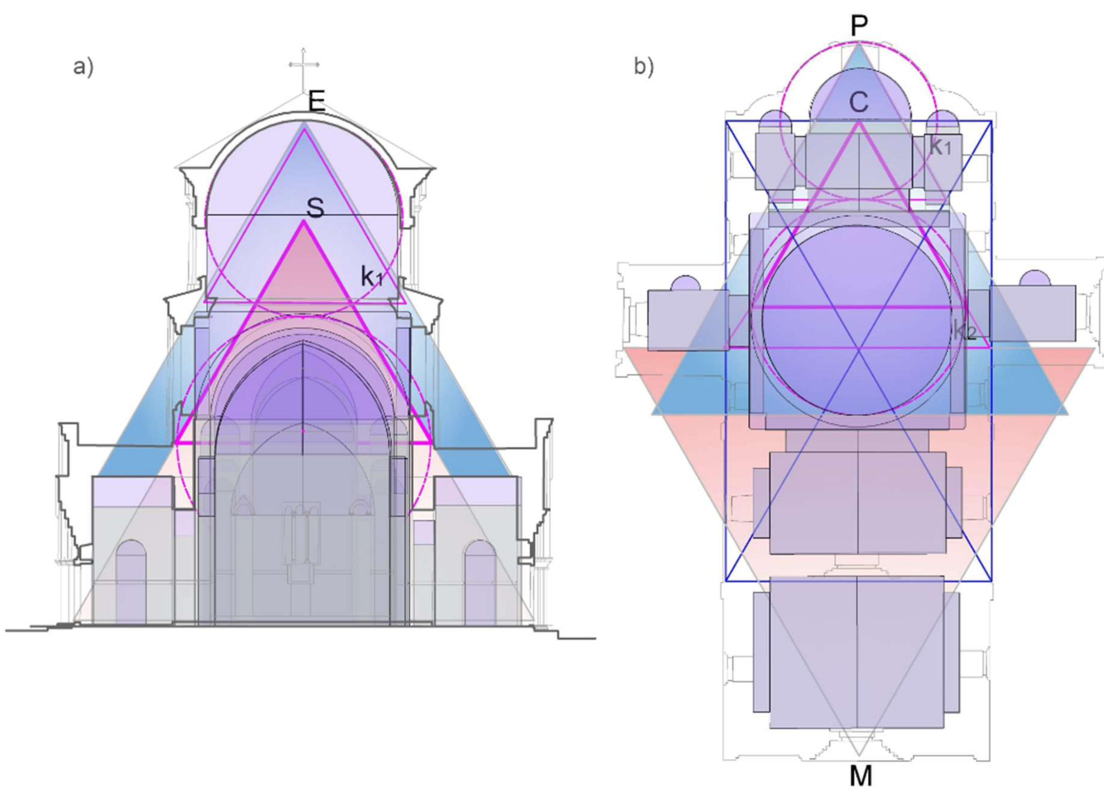

Fig. 12 Representation of the 3D structural model accuracy of the Studenica monastery church by its proportional scheme: a) cross section, b) floor plan.

This part of the research shows how the variations of the two critical points of the church interior, the centres of the dome and of the main apse, along with the key triangles and the rectangle (1:√3), incorporated in the proportional scheme, affect the final 3D structural model.

Conclusion

The 2D geometric proportional analysis of the three selected Serbian medieval churches points to the potential use of sophisticated triangular proportioning in the architectural design of their layouts and cross-sections. Although the builders were not skilled enough to obtain exact right angles and symmetry in the layout of the walls, the detailed geometric analysis presented here reveals the regular geometric schemas behind the original architectural design. Geometric schemas based on the circle and equilateral triangle are clearly related to the architectural design of all three analysed churches.

Based on the detailed analytical drawings for the church layouts and cross-sections (through the central space and the dome), geometric shapes of the circle and equilateral triangle are incorporated into the proportional scheme, as elaborated in Štambuk's canon. According to Štambuk, two circles are constrained by an equilateral triangle, which he calls *the holy* triangle; the smaller circle overlaps the circumscribed circle of the main apse, and it is tangent to the larger circle; the *holy* triangle has its top in the centre of the apse; the larger circle diameter is equal to the edge of the *holy* triangle. The edge of the triangle corresponds to the width of the church nave. The common tangent of the two circles divides the most sacred altar space from the nave. This canon is used for the first time in analyses of the medieval churches of the Raška group.

The main contribution of this work is that it reveals the two correspondent proportional schemas (the scheme of the layout and the cross-section through the dome of the church) consisting of three main triangles, two circles, one rectangle (with ratio 1: √3) and one minor triangle. These interrelated geometric shapes are determined by the characteristic points in the layout and the cross-section of the church. In Štambuk's construction, they are the centre of the circle of the apse and the centre of the dome, respectively. The newly recognised characteristic points of the geometric schemas are the centre of the western entry wall of the church and the apex point of the outer contour of the apse in the layout, as well as the highest point of the outer contour of the dome in the cross-section. This advanced proportional scheme is presented in three variants S, Z, and G. In accordance with the differences in the programme of the churches, additional equilateral triangles appear, which determine the position of the side chapels. The concepts of these schemas and the relationships of the geometric shapes with the dimensions of the analysed churches are:

- Štambuk's holy triangle has the critical role in the disposition of the scheme. The top of the triangle is in the most sacred place, at the altar table (oriented towards the east) and the centre of the dome, in the layout and cross-section respectively. The sides of the angles determine the width of the altar's central arch, or the lateral arches of the altar (variant G).
- The holy triangle is in direct correlation with the rectangle 1: √3. The midpoint of the shorter, eastern side of the rectangle coincides with the triangle's vertex. The rectangle, in its general concept, determines the dimensions of the nave of the church (without the semi-circular part of the main apse).
- Placed at the centre of the western, entrance wall of the church in the layout of the church, the second most important equilateral triangle determines the total width of the church (together with the choirs). The side of the triangle against its vertex coincides with the contour of the western wall of the central core. A triangle of the same size, in the cross-section through the dome of the church, pointed upwards, determines the position of the centre of the dome, that is, the base of the tambour (variant Z).

- The third equilateral triangle, with its vertex at the highest point of the main apse of the layout of the church, defines the total inner width of the church. A triangle of the same dimensions, in the cross-section through the dome of the church, defines the height from the side vaults to the top of the dome.
- A smaller equilateral triangle defines the internal dimensions of the altar space, that is, the dome together with the tambour, in the layout and cross-section, respectively.

The observed regularities, despite slight differences (related to either the interior or exterior dimensions of the walls), lead to the assumption that other churches within the Raška group may have been designed according to the same principles. As such, they provide the possibility of the potential restoration of these or similar objects. The proportional schemas S, Z and G, which derive from the Štambuk canon, play a key role in defining the dimensions of all elements of the structure of the church in the layout, in particular, and other specific elements in the cross-sections, such as the vaults of the lateral chambers, the tambour and dome, the altar arches and the arches of the central core. These proportional schemas S, Z and G have the potential to be parametrically set, and used for the examination of other Raška churches.

Additional results of this research are the 3D models of the interior structures of the Studenica, Žiča and Gradac monasteries. These models consist of the elements of the parametric library, i.e. characteristic parts of the church structure (arch, vault, apse, drum, dome, etc.), created in Autodesk Revit software. Based on a particular profile geometry, their spatial dimensions, and proportions, the models were interactively used with the corresponding proportional schemas. Beside the potential use of the parametric elements of the church structure for the modelling of other Raška religious structures, the 3D models offer a more comprehensive understanding of the geometric complexity in the design of medieval architecture and call for further investigations of the parametric modelling relevant for current HBIM initiatives.

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